Implementation of tomography based on TOMO2

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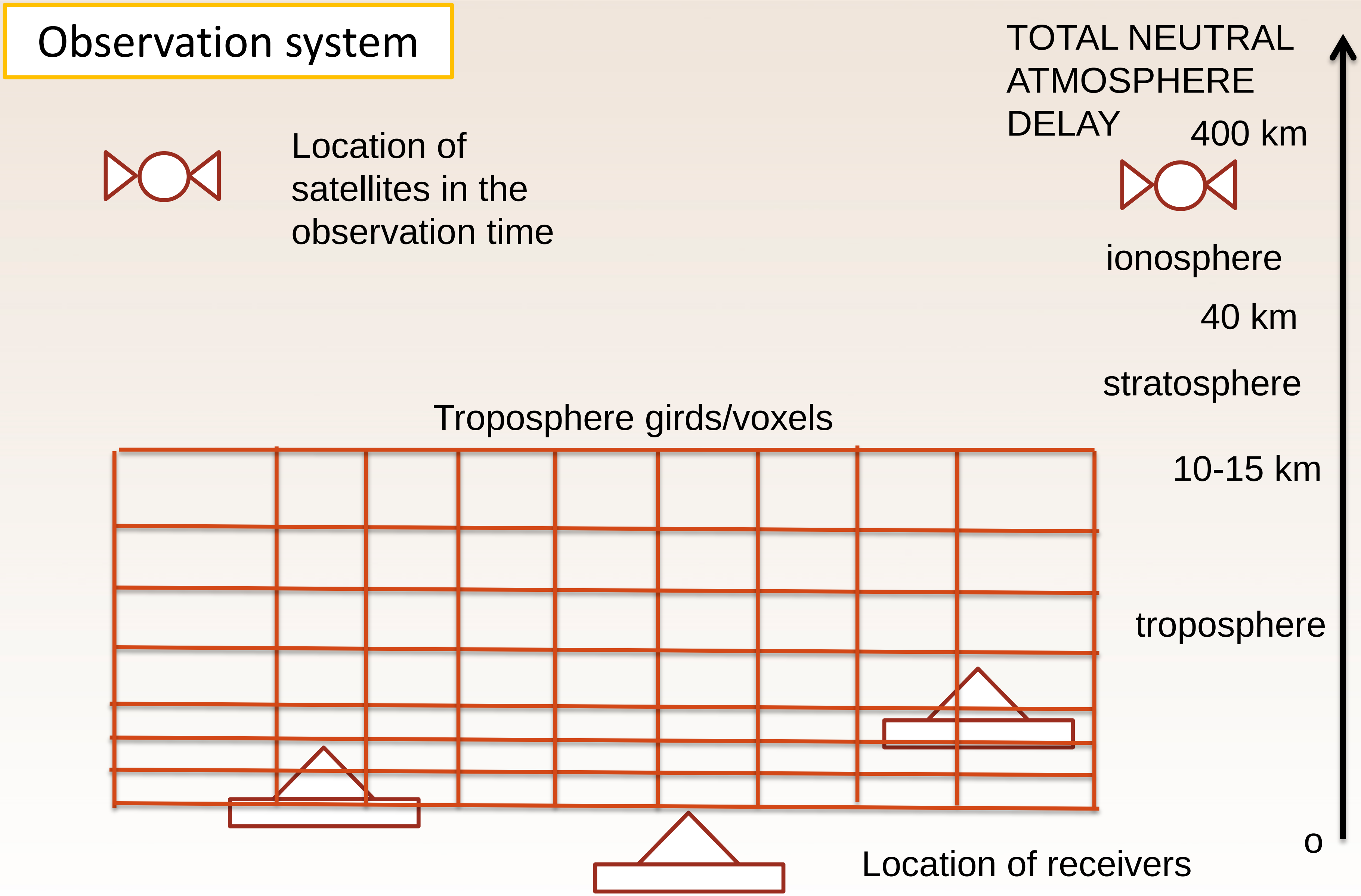
Tomography: structure



Tomography: observations (1)

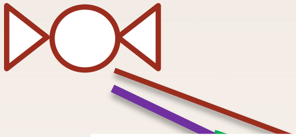
Observations

$$STD = \delta\rho_{apr,R}(z_R^S) + \delta^h\rho_R(t)\cdot mf_N(z_R^S) + \delta^n\rho_R(t)\cdot\frac{\partial mf_N}{\partial z}\cos A_R^S + \delta^e\rho_R(t)\cdot\frac{\partial mf_N}{\partial z}\sin A_R^S + ZDres$$

ZTD

ZTD Gradients

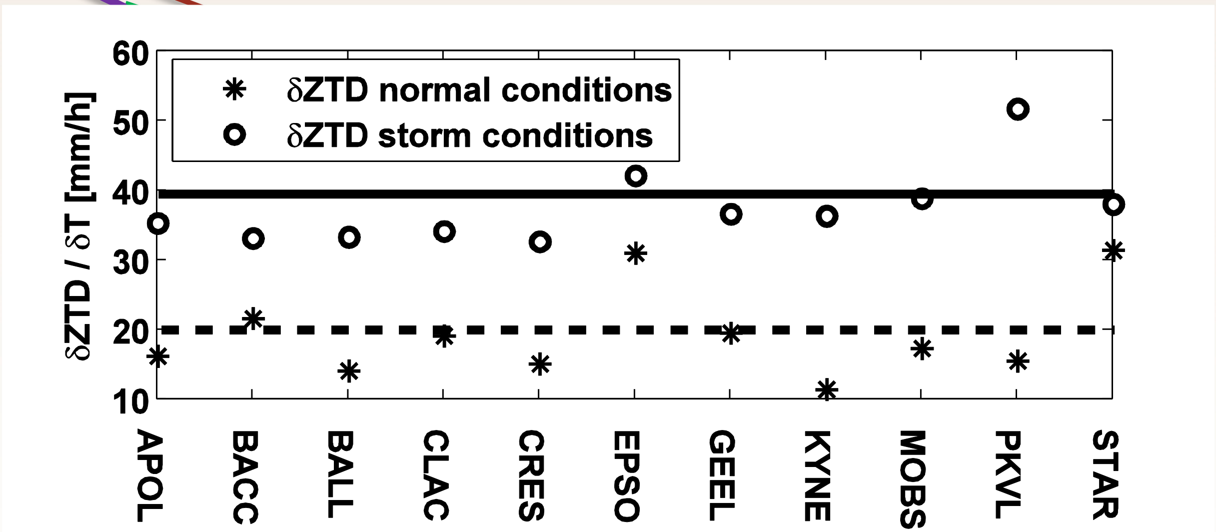
ZD Residuals



Stochastic modelling

Functional modelling

$$\frac{d\delta^h\rho_R}{dt} = \frac{\delta^h\rho_R}{t} + w(t)$$



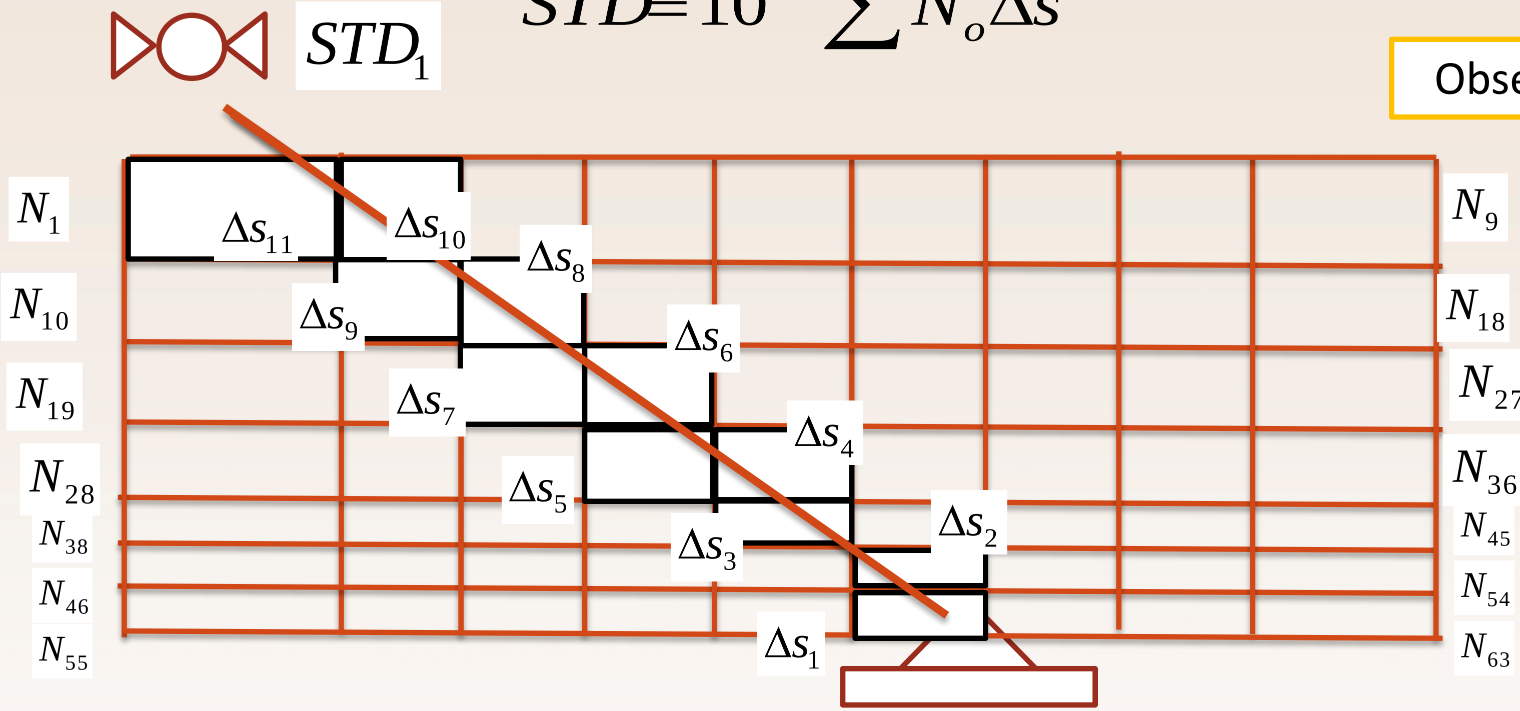
Tomography: single observation (2)

Observations

Link

Observation system

$$STD = 10^{-6} \sum N_o \Delta s$$



$$STD = 10^{-6} \cdot (N_{60} \cdot \Delta s_1 + N_{51} \cdot \Delta s_2 + N_{41} \cdot \Delta s_3 + N_{32} \cdot \Delta s_4 + N_{31} \cdot \Delta s_5 + N_{22} \cdot \Delta s_6 + N_{21} \cdot \Delta s_7 + \ldots.$$

$$\ldots. + N_{12} \cdot \Delta s_8 + N_{11} \cdot \Delta s_9 + N_2 \cdot \Delta s_{10} + N_1 \cdot \Delta s_{11})$$

Tomography: multiple observations (3)

Observations

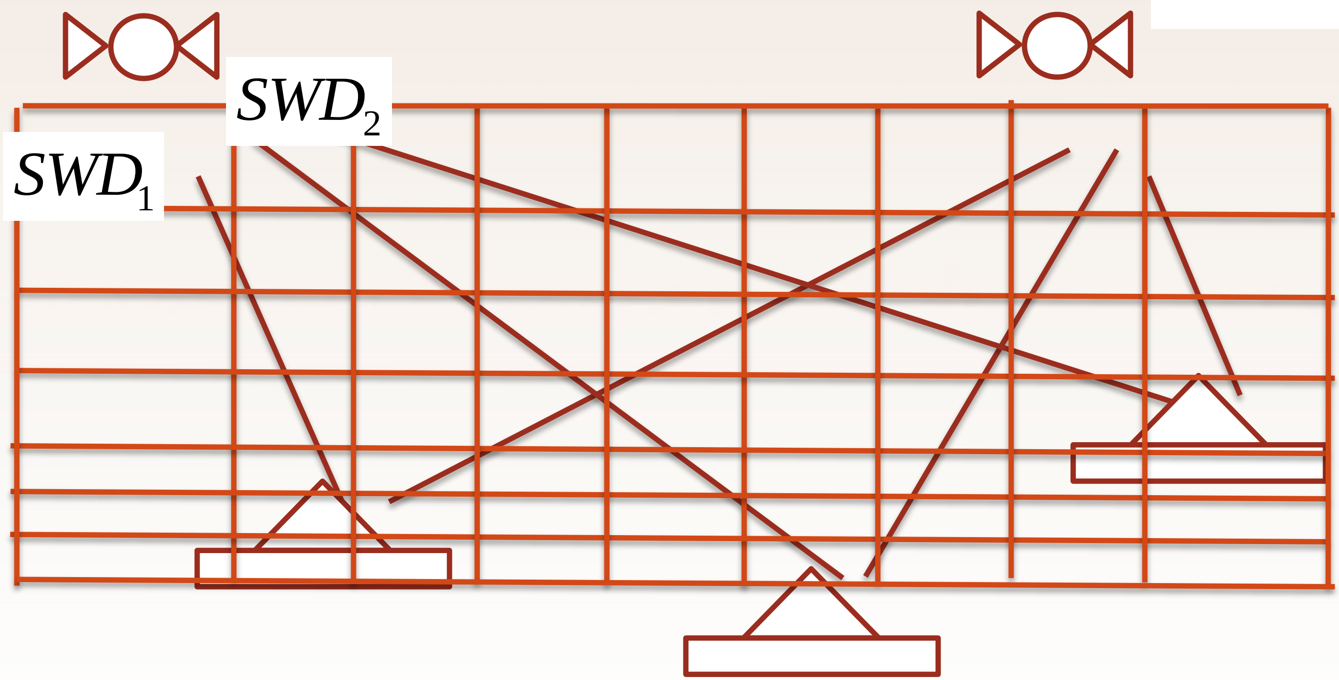
$$SWD = \begin{bmatrix} SWD_1 \\ SWD_2 \\ \vdots \\ SWD_n \end{bmatrix}$$

Tomography model structure

$$A = \begin{bmatrix} d_{11} & 0 & 0 & 0 & \cdots & d_{1m} \\ d_{21} & d_{22} & d_{23} & 0 & \cdots & d_{2m} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ d_{n1} & d_{n2} & 0 & d_{n4} & \cdots & d_{nm} \end{bmatrix}$$

Unknowns

$$N_w = \begin{bmatrix} N_{w1} \\ N_{w2} \\ \vdots \\ N_{wm} \end{bmatrix}$$



Tomography: problem ill-posedness

EXAMPLE POLAND
120 ZTDs every hour ~600 SWDs ,
Number of unknowns (10x12x10) =
1200 voxels
A matrix is sparse
SWDs are correlated

$$SWD = A \cdot N_v$$

$$[600x1] = [600x1200] \cdot [1200x600]$$

Solve the system:

$$N_v = (A^T A)^{-1} A^T SWD$$

Add weights (P) and constraints (B)

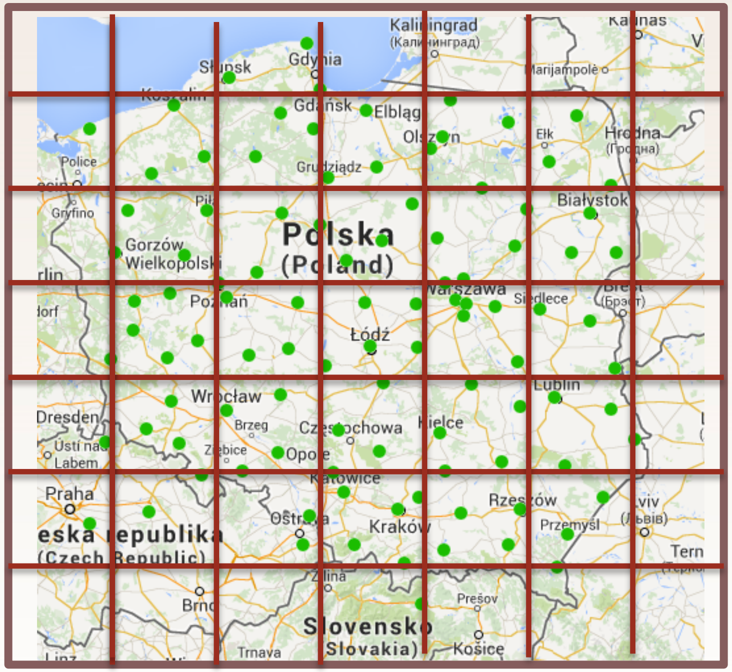
$$N_v = (A^T \cdot P \cdot A + B^T \cdot B)^{-1} A^T \cdot P \cdot SWD$$

Use pseudo inverse not unique but optimal (SVD)

$$N_v = (A^T \cdot P \cdot A + B^T \cdot B)^{+} A^T \cdot P \cdot SWD$$

Select best singular values with (TSVD)

$$N_v = (A^T \cdot P \cdot A + B^T \cdot B)^{+} A^T \cdot P \cdot SWD$$



Tomography: constraints(1)

Observations

$$SWD = \begin{bmatrix} SWD_1 \\ SWD_2 \\ \vdots \\ SWD_n \end{bmatrix}$$

$$w_{ij} = dist(voxel_i, voxel_j)$$

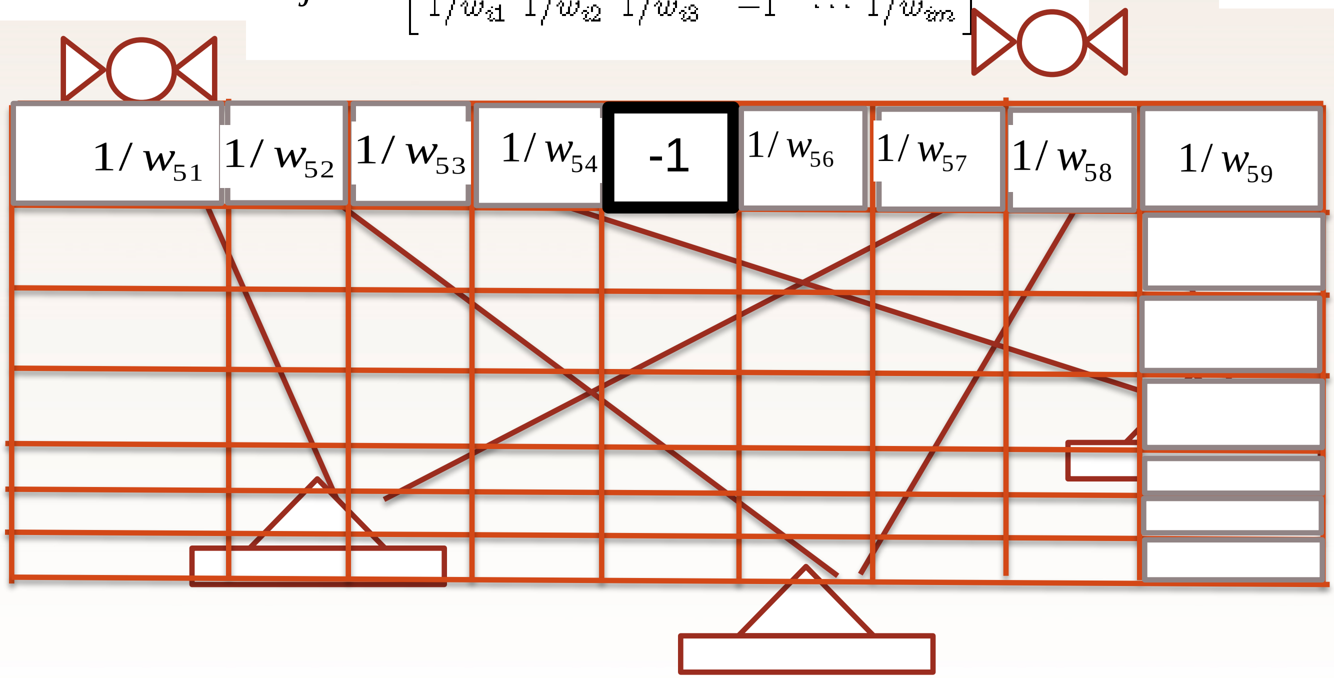
Tomography model structure

$$A = \begin{bmatrix} d_{11} & 0 & 0 & 0 & \cdots & d_{1m} \\ d_{21} & d_{22} & d_{23} & 0 & \cdots & d_{2m} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ d_{n1} & d_{n2} & 0 & d_{n4} & \cdots & d_{nm} \\ -1 & 1/w_{12} & 1/w_{13} & 1/w_{14} & \cdots & 1/w_{1m} \\ 1/w_{21} & -1 & 1/w_{23} & 1/w_{24} & \cdots & 1/w_{2m} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1/w_{i1} & 1/w_{i2} & 1/w_{i3} & -1 & \cdots & 1/w_{im} \end{bmatrix},$$

Unknowns

$$N_w = \begin{bmatrix} N_{w1} \\ N_{w2} \\ \vdots \\ N_{wm} \end{bmatrix}$$

Horizontal constraints



Vertical constraints

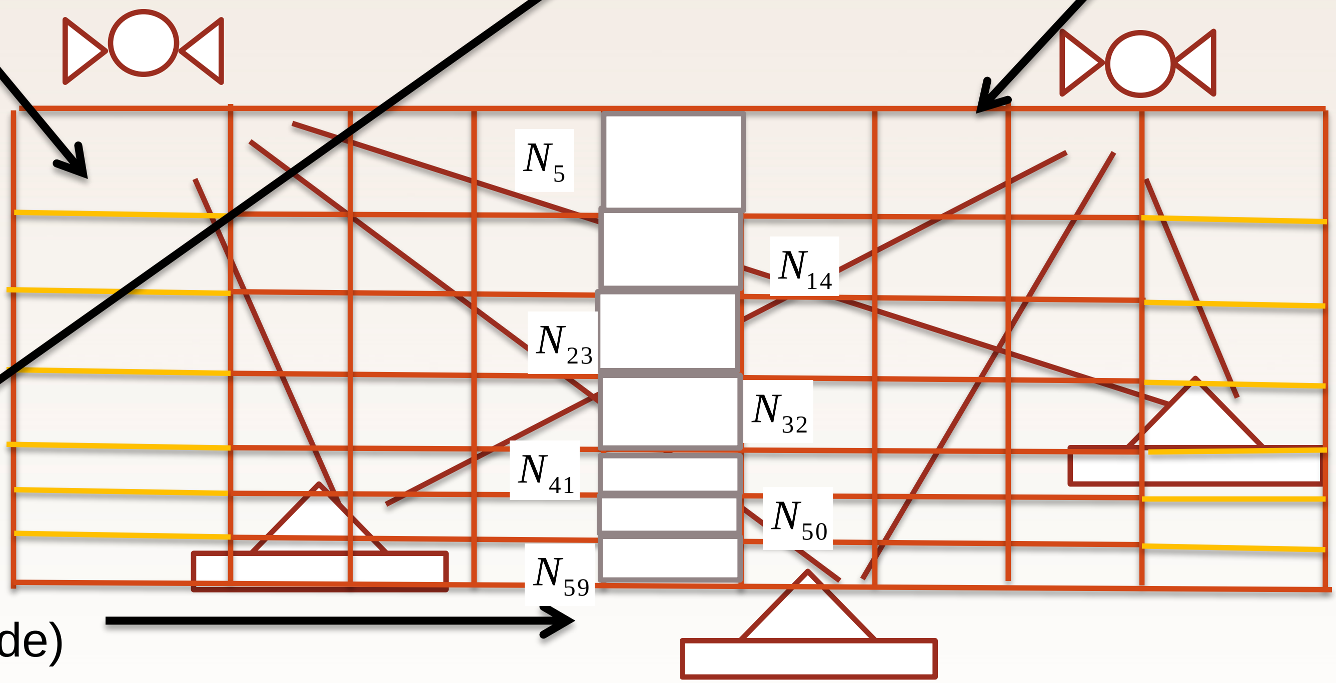
Tomography: pseudo observations and outer (1)

Observations | Tomography model structure | Unknowns

$$\begin{bmatrix} \mathrm{SWD} \\ \mathrm{N}_{\mathrm{vapriori}} \end{bmatrix} = \begin{bmatrix} A_{inner} & A_{outer} \\ A_{apriori} & 0 \end{bmatrix} \begin{bmatrix} N_{v} \\ No_{v} \end{bmatrix} + v$$

Outer model domain

Inner model domain



Observations (e.g. radiosonde)

Tomography implementations: KF/RKF

$$\begin{bmatrix} SWD \\ N_{waprioir} \\ 0 \end{bmatrix} = \begin{bmatrix} A \\ A_{apriori} \\ W \end{bmatrix} \cdot N_w$$

(Rohm et al., 2014)

I

State first guess

State predicition

$$\hat{N}w_k(-) = \Phi_k \cdot \hat{N}w_{k-1}(+)$$

Corrected state estimate

$$\hat{N}w_k(+) = \hat{N}w_k(-) + K_k \cdot \left(SWD_k - A_k \cdot \hat{N}w_k(-)\right)$$

Predicted

$$Q_k(-) = \Phi_k \cdot Q_{k-1}(+) \cdot \Phi_k^T + Nproc_{k-1}$$

Process noise

Corrected state covariance

$$Q_k(+) = Q_k(-) - K_k \cdot A_k \cdot Q_k(-)$$

Kalman Gain matrix (Robust KF iterative process)

$$K_k = Q_k($$

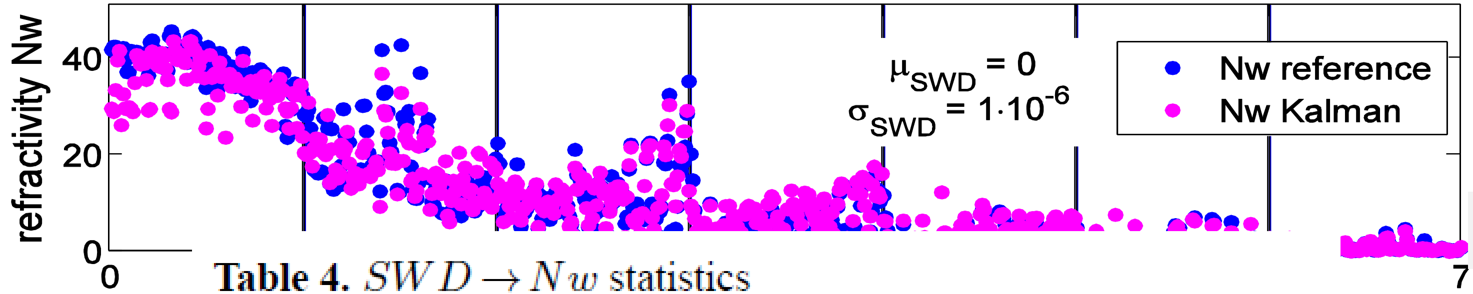
Design matrix

Inversion process

Observation noise
Robust KF - modification

Tomography retrieval quality

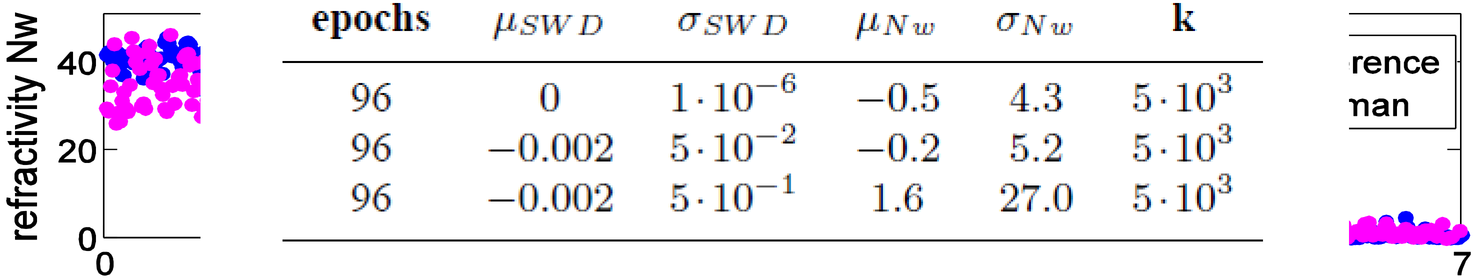
Robust Kalman Filter



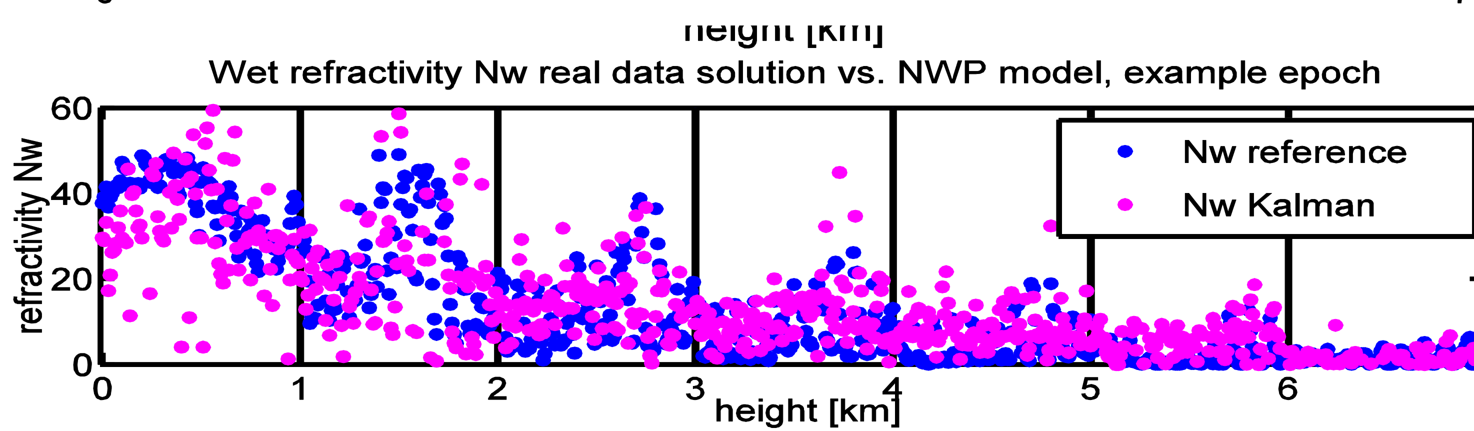
zero noise

Table 4. $SWD \to Nw$ statistics

epochs	μ_{SWD}	σ_{SWD}	μ_{Nw}	σ_{Nw}	k
96	0	$1 \cdot 10^{-6}$	-0.5	4.3	$5 \cdot 10^{3}$
96	-0.002	$5 \cdot 10^{-2}$	-0.2	5.2	$5 \cdot 10^{3}$
96	-0.002	$5 \cdot 10^{-1}$	1.6	27.0	$5 \cdot 10^{3}$

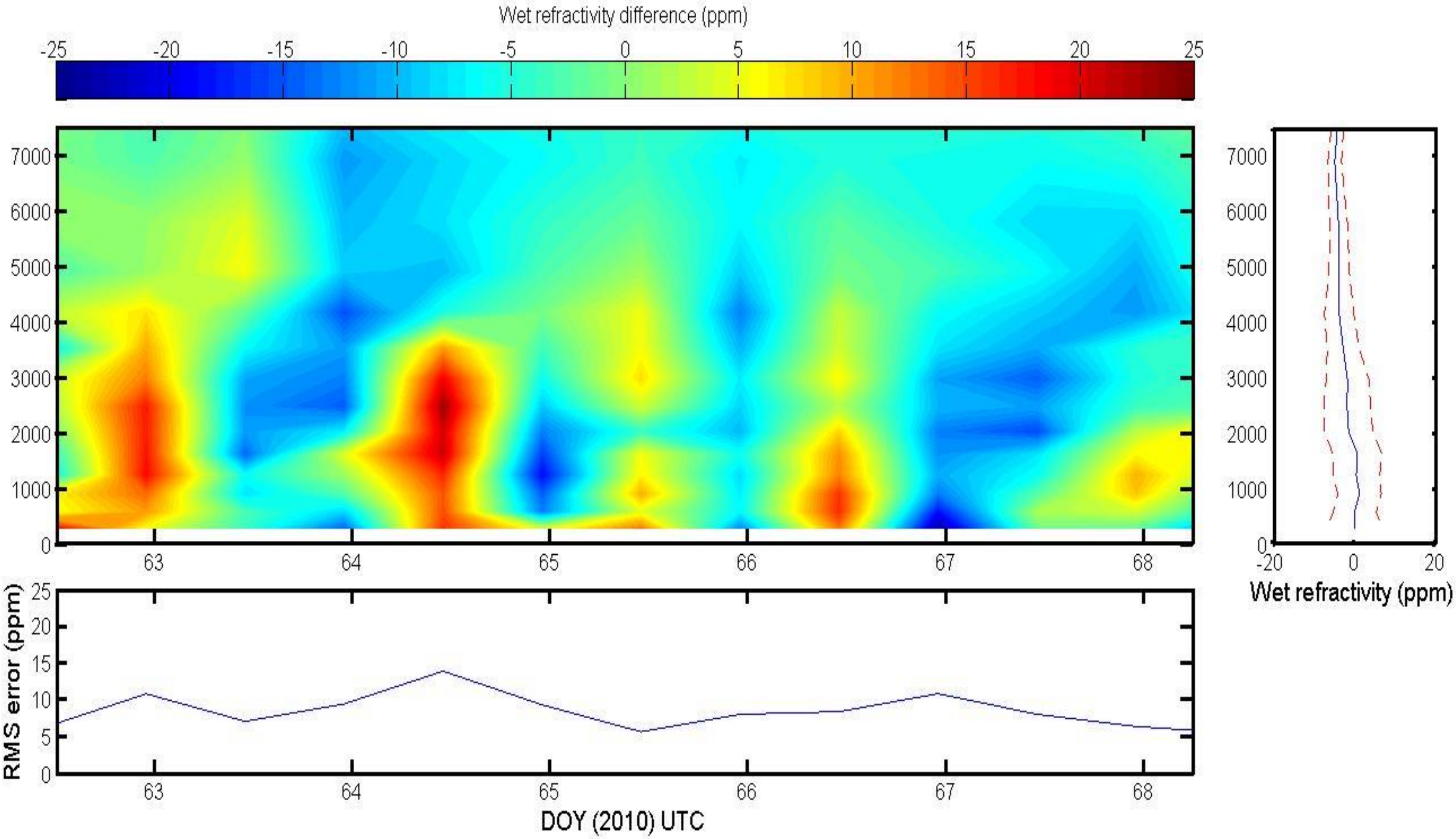


low noise

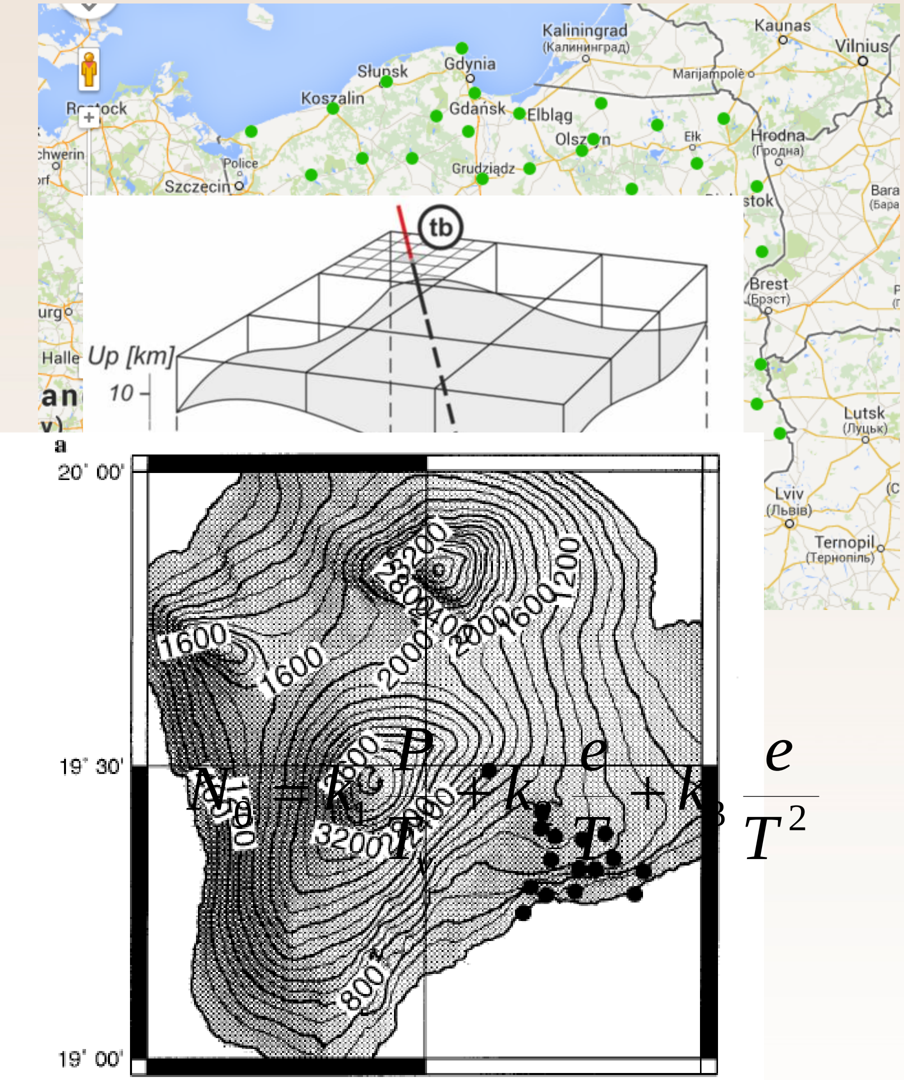


noise reduction

Tomography retrieval quality



Tomography: practical considerations



$$N_0 = k_1 \frac{P}{T_v} + k_2 \frac{e}{T} + k_3 \frac{e}{T^2}$$

Flores, A., Ruffini, G., & Rius, A. (2000, February). 4D tropospheric tomography using GPS slant wet delays. In *Annales Geophysicae* (Vol. 18, No. 2, pp. 223-234). Springer-Verlag.

ZTD to SWD conversion supported with pressure information – NWP is a reasonable choice of pressure data

The size of the voxels should not be smaller than half the distance between stations

Heavy undulated areas are better for tomography.

A matrix condition number is a good approximation of the tomography geometry quality

$$cond(A) = \frac{s_k}{s_1}$$

Summary

- The tomography is a technique to convert ANY 1D observations to 3D structure
- GNSS tomography is based on: 1) the Slant Troposphere observations, 2) division of the troposphere into number of voxels and 3)know link between troposphere conditions and signal propagation
- GNSS tomography implementation for troposphere studies should resolve ill-posedness of the observation system
- The quality of retrieval depends on the interstation distance, terrain undulation, available independent observations.
- There is potential to use it in both Nowcasting and NWP and we are very keen to work with you on those applications

Thank you!

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